

## The quantum state diffusion picture of physical processes

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1993 J. Phys. A: Math. Gen. 26 2245

(<http://iopscience.iop.org/0305-4470/26/9/019>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.68

The article was downloaded on 01/06/2010 at 21:16

Please note that [terms and conditions apply](#).

# The quantum state diffusion picture of physical processes

Nicolas Gisin† and Ian C Percival‡

† Group of Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland

‡ Department of Physics, Queen Mary and Westfield College, University of London, Mile End Road, London E1 4NS, UK

Received 8 December 1992

**Abstract.** In earlier papers we introduced a quantum state diffusion model for the evolution of an individual open quantum system, and proved localization theorems based on this model. This paper shows in more detail how the diffusion leads to localization in position and phase space, and to symmetry breaking for chiral molecules. The theory of radioactive decay of absorbers and detectors is described in the state diffusion picture. The Mott and Gurney theory of latent image formation in photography is presented in its state diffusion version. It is an example of quantum detection without significant amplification.

## 1. Introduction

In [1] we introduced a state diffusion model for the state of an open quantum system, and showed how this model could be applied to investigate simple physical processes. We emphasized the practicality of the model for computation. In the preceding paper in this issue, we proved theorems on localization [2]. In this paper we show how the model can be applied to a wide variety of physical processes in which localization plays an important role. The emphasis is on the physical insight which is achieved in representing the stochastic behaviour of an individual open quantum system explicitly. This contrasts with the usual representation of the averaged properties of an ensemble by density operators, but is consistent with it.

The notation, conventions, definitions and theorems of the previous paper [2] in this issue will be followed throughout, and that paper should be used to refer to details that are not given here.

Instead of looking at the deterministic evolution of the density operator  $\rho$  representing an ensemble of systems  $A$ ,

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \sum_m (L_m \rho L_m^\dagger - \frac{1}{2} L_m^\dagger L_m \rho - \frac{1}{2} \rho L_m^\dagger L_m) \quad (1.1)$$

we look at the stochastic diffusion of a quantum state  $|\psi\rangle$  representing an individual system of the ensemble in interaction with its environment  $B$ . The corresponding quantum state diffusion equation is a stochastic differential equation for the normalized state vector  $|\psi\rangle$ , whose differential Itô form is, from [2],

$$d|\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle dt + \sum_m (\langle L_m^\dagger \rangle_\psi L_m - \frac{1}{2} L_m^\dagger L_m - \frac{1}{2} \langle L_m^\dagger \rangle_\psi \langle L_m \rangle_\psi) |\psi\rangle dt + \sum_m (L_m - \langle L_m \rangle_\psi) |\psi\rangle d\xi_m. \quad (1.2)$$

This difference is particularly clear in the theories of localization and measurement. In this picture localization (or reduction) in  $A$  is a stochastic process caused by interaction of the system with its environment, which may or may not be a measuring apparatus. A partition of the state space of  $A$  divides it into orthogonal subspaces or channels with projectors  $P_k$ . Where ensemble probabilities in these channels are preserved during the interaction, the individual systems tend to become localized in the channels in the sense that the mean square deviation or dispersion entropy is reduced, with an end result that is extremely close to a *complete* localization, with each system of the ensemble confined to one channel, an ideal limit of the real incomplete localization. For continuous dynamical variables, like the very important case of position, the localization is never complete in this sense.

In section 2 we introduce a helpful analogy between elementary circuit theory and localization theory, with particular reference to localization in position space. In section 3 we find that state diffusion due to the environment breaks the chiral symmetry of molecules, as an elementary consequence of position localization, and the interaction of any symmetric system with any fluctuating environment breaks the symmetry of the the system.

Section 4 describes the quartic oscillator or double-well model, and sample solutions of this model are used to illustrate some of the general principles that apply to more complicated systems like molecules, and also the phase-space localization of macroscopic systems, which is discussed in section 5.

In section 6, the state diffusion theory is applied to an absorbing screen and to a detector, leading to section 7, in which the state diffusion model of latent image formation in a photographic plate is described. The last example is the radioactive decay described in section 8, and we finish with a discussion of the role of state diffusion in the final section 9.

## 2. Electric circuits and position localization

There is a close analogy between the state diffusion theory of quantum mechanics and classical circuit theory. An electric circuit includes both Hamiltonian elements, with capacitance and inductance, and non-Hamiltonian elements, such as resistors, which represent the interaction of the system with its environment. This interaction produces stochastic fluctuations in the form of thermal noise, which could be represented by an Itô equation. The resistors are treated on the same basis as the Hamiltonian parts of the system, and it is not considered necessary to analyse the detailed physics of every resistor before attempting to solve a problem in circuit theory. Nevertheless, it is important that such an analysis should be possible, as, for example, when treating an aerial as a resistor.

Similarly the environment operators of the state diffusion theory represent particular types of interaction of the system with its environment. There is no need to carry out a detailed physical analysis of these interactions before attempting to solve a problem in the theory of open quantum systems, as shown in [1]. In particular each channel, consisting of any number of states, can be represented by a one-state channel. The transitions between channels are then represented by relatively simple operators. For example annihilation and downward transition operators can represent dissipation by the environment, and localization in eigenspaces of a dynamical variable  $x$  can be represented by an environment operator  $X$ . If the operators are chosen well, then localization of quite complicated systems can be represented adequately if approximately by these simple models.

From [2], position localization is particularly common and important because interactions are localized in position. There is very approximate localization on the

eigenstates of the position operator, but perfect localization is impossible in this case, both because interactions are not perfectly localized in position, and because it would require infinite energy to produce this pure state.

Usually the interactions are complicated but, in the spirit of circuit theory, the effect of the localization on the system can be approximated by relatively simple environment operators, of which the simplest is the position operator itself, with a constant factor. This is used in section 5 on phase-space localization. It might then be helpful to think of the environment as performing a measurement of position. But this is just a way of thinking about the localization, because most environments do nothing of the kind!

A better approximation, which is still fairly simple, is to represent the local interactions by a set of operators  $P_k$  which project onto the regions of a partition of position space, and a further improvement is to multiply each projection operator by other operators which represent the details of the interaction in that region, such as annihilation operators of a local linear oscillator, which represent dissipation. These are local operators in the sense of [2]. This is applied to the double well in section 4.

For compound systems with many particles, the interaction is *not* localized in the configuration space of the system, but the interaction of each distinguishable particle with the environment is localized in position space. So the overall effect is to localize each particle in position space, which is equivalent to localizing the whole system in configuration space.

Now consider identical particles. In the state diffusion equation (1.2) of [2], the environment operators  $L_m$  operating in the space of  $A$ -states are matrix elements for a basis in the space of  $B$ -states of the Hamiltonian  $H_{AB}$  for the system and environment together. If the basis has the correct symmetry, then these matrix elements are invariant under permutation of the particles, whether they are bosons or fermions. So the right-hand side has the same fermion or boson symmetry properties as the state vector, and the diffusion equations preserve the permutation symmetry of the state vector. Therefore indistinguishable particles cannot be localized individually, but the effect of the interaction is to localize in a particular configuration, together with those other configurations obtained by permutation of the particles. Bell [3] has remarked that the original quantum jump position localization theory of Ghirardi *et al* [4] destroyed the symmetry of fermion and boson state vectors, and this fault was remedied by the corresponding state diffusion theory, as remarked by Gisin [5], Ghirardi *et al* [6] and by Diósi [7].

### 3. Symmetry breaking and chirality

When the Schrödinger equation for a pure state or the Bloch equation (1.1) for a density operator are invariant under a symmetry operation, then there is no explicit mechanism for the destruction of the symmetry. But the state diffusion equations (1.2) are stochastic, so if individual states of the environment are not symmetric, individual systems diffuse away from an initial symmetric state, and the symmetry of the state is broken. In state diffusion theory this is no more surprising than the broken spherical symmetry of the path of a Brownian particle diffusing away from some fixed origin in space.

In the case of ammonia, the molecule has symmetry with respect to inversion in a plane parallel to the plane of the hydrogen atoms, but unlike the permutation symmetry of particles, the environment does not respect this symmetry, so in this case the localization produced by the diffusion can and does destroy the symmetry for individual systems, even though the symmetry of the ensemble is preserved. This is general.

In the very common case of molecules in a sufficiently dense environment of atoms or other molecules, the configuration localization of the previous section results in a molecule which is localized in the neighbourhood of a definite configuration. For the ammonia molecule, this destroys the reflection symmetry, and for chiral molecules it selects one of a pair of isomers which transform into each other under reflection in a plane. So the breaking of chiral symmetry is a direct and elementary consequence of position localization, which is a consequence of the locality of interactions in position space.

The principles of this symmetry breaking are illustrated by the double-well model of section 4.

The suggestion that the environment can break chiral symmetry has a long history [8–11]. When the evolution of a chiral system is described by a density operator, then, like the other forms of localization, treated in this way, there is no explicit representation of the process that leads to the chirality of individual systems. It is inferred from the reduction in the off-diagonal matrix elements of the density matrix. Nevertheless, this work is very valuable for the quantum state diffusion theory, since it gives a detailed account of the physical processes that lead to the symmetry breaking. According to state diffusion theory, the breaking of the symmetry of chiral molecules is just one more example of the power of position localization, which leads to configuration localization, which in turn is incompatible with the symmetric states.

For the inverse process, the remarkable result quoted by Harris and Stodolsky [12] that at low temperatures ‘the more rapid the rate of collisions the less rapidly the chirality can relax’ is because the diffusion localization dominates the barrier penetration. The asymmetry of the environment leads directly through its interaction with the system to maintain the asymmetry of the states of the individual systems of the ensemble.

#### 4. The double-well model

This is a convenient model to illustrate some of the principles of localization.

The Hamiltonian is

$$H = \frac{1}{2}p^2 - \frac{1}{4}q^2 + \frac{1}{8\ell^2}q^4 \quad (4.1)$$

giving the double-well potential illustrated in figure 1 for  $\ell = 8$ . The energy eigenstates are even  $|en\rangle$  and odd  $|on\rangle$ , each sequence labelled in order of increasing energy. In the neighbourhood of the minima at  $\pm\ell$  the potential looks like a well with angular frequency  $\omega = 1$ . With appropriate phase conventions, the left (–) and right (+) states of the double well are linear combinations of the even and odd states, given by

$$\begin{aligned} | - n \rangle &= \frac{1}{\sqrt{2}}(|en\rangle + |on\rangle) \\ | + n \rangle &= \frac{1}{\sqrt{2}}(|en\rangle - |on\rangle). \end{aligned} \quad (4.2)$$

For low values of  $n$ , the pairs of even and odd states are nearly degenerate, and the left and right states are excellent approximations to the eigenstates of the left and right oscillators of the double-well. For these states the coupling between the left and right oscillators can be neglected in most environments, and the Hamiltonian is almost the same as that of a single

oscillator with unit angular frequency together with a degenerate spin- $\frac{1}{2}$  system. Before treating all the details of the double-well potential, it is useful to start with this simple model, whose Hamiltonian is

$$\begin{aligned}
 H_a &= \frac{1}{2}(p^2 + q^2) \\
 &= \frac{1}{2}(p_+^2 + q_+^2) + \frac{1}{2}(p_-^2 + q_-^2)
 \end{aligned}
 \tag{4.3}$$

where, by definition,

$$p_{\pm} = P_{\pm}p \quad q_{\pm} = P_{\pm}q
 \tag{4.4}$$

and  $P_+$  and  $P_-$  are projection operators onto the  $+z$  and  $-z$  states of the spin. The second form for  $H_a$  illustrates explicitly a general expression for separability which can be generalized to a wide class of localization and measurement problems.

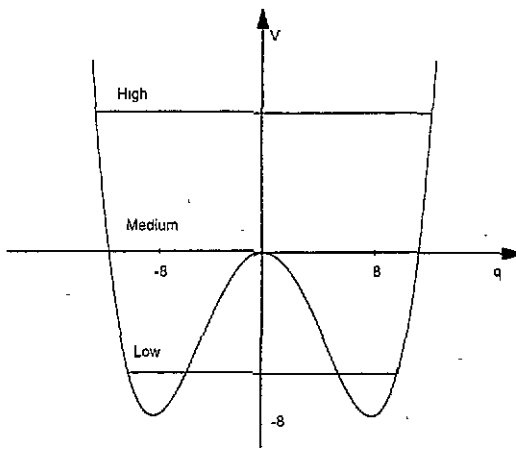


Figure 1. The quartic double-well potential (4.1) with  $\ell = 8$ . Horizontal lines represent the low, medium and high initial mean energy of section 5.

Interaction with the environment can take place at either well. We suppose that these interactions are sufficiently localized that they operate on the left well or the right well, but not on both. In that case they can be represented by environment operators which can also be split into two types, and which act in the subspaces defined by  $P_+$  and  $P_-$ , so that they are of the form  $L_{m+}$  or  $L_{m-}$ , where

$$\begin{aligned}
 L_{m+} &= P_+L_{m+} = L_{m+}P_+ \\
 L_{m-} &= P_-L_{m-} = L_{m-}P_-
 \end{aligned}
 \tag{4.5}$$

Figure 2 presents a sample solution of the diffusion equation for the case of two environment operators

$$L_{\pm} = P_{\pm}a
 \tag{4.6}$$

where  $a$  is a standard annihilation operator. The initial state is a linear combination of coherent states with equal amplitude for each oscillator. It is an approximation to a linear

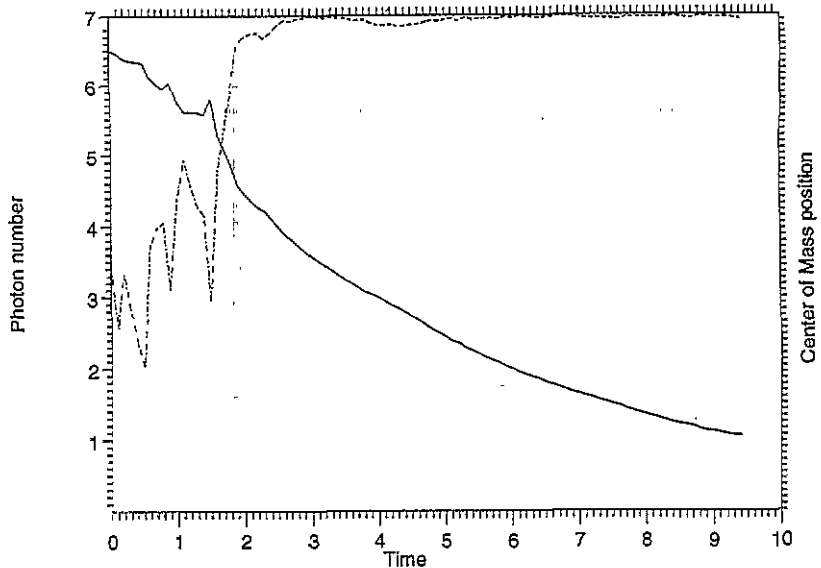


Figure 2. Single run of the spin- $\frac{1}{2}$  simplified double-well model, with Hamiltonian (4.3) and two dissipative environment operators (4.6). The graphs are of mean spin  $\langle \sigma_x \rangle$  and mean photon number  $\langle a^\dagger a \rangle$  in each well as a function of time. The first shows the localization in the right well, denoted +, and the second shows the effect of the dissipation, with reduced fluctuations at later times.

combination of coherent states with the same amplitude in each well for the original quartic oscillator. After a transient time with large fluctuations, the system stabilizes into one well or the other. Another sample would localize after following a different random path. The relative frequency of localization in one well or the other is equal to the weight of the initial state in that well. This is a consequence of the consistency of the diffusion equations with the usual density operator theory.

We now come back to the real quartic double well in the presence of decay and of separate interaction of the two halves of the well with the environment. In order to simulate this interaction we extended the computer program of [1] to include the operator

$$R = \sum_{n \geq 0} (|2n\rangle\langle 2n+1| + |2n+1\rangle\langle 2n|). \quad (4.7)$$

Here  $|n\rangle$  is the  $n$ th excited state of a harmonic oscillator centred at the origin, where there is a local potential maximum of the double well. The projectors onto the left ( $P_-$ ) and right ( $P_+$ ) halves of position space are then well approximated by the following projectors:

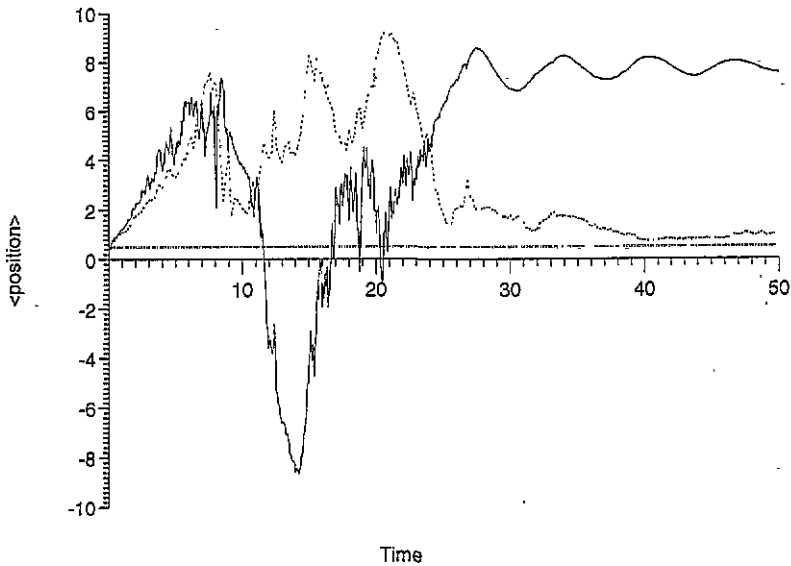
$$P_{\pm} = \frac{1}{2}(1 \pm R). \quad (4.8)$$

The annihilation operator on the left well of the double oscillator, representing a dissipative interaction with that well only, is given by

$$a_- = (1/\sqrt{2})P_-(q + \ell + ip)P_- = P_-(a + \ell/\sqrt{2})P_-. \quad (4.9)$$

Similarly

$$a_+ = P_+(a - \ell/\sqrt{2})P_+. \quad (4.10)$$



**Figure 3.** Symmetry breaking for the quartic double well (4.1) with two dissipative environment operators given by (4.11). The plots for a single run show the mean position  $\langle q \rangle$  and the RMS deviation in position  $\Delta q$ . The localization in one well and the reduction in the variation are clearly shown.

Figure 3 illustrates symmetry breaking for the double-well potential (4.1) with  $\ell = 8$  and two dissipation environment operators

$$L_- = 2^{-3/2}a_- \quad L_+ = 2^{-3/2}a_+ \quad (4.11)$$

representing independent dissipative interactions with the environment for each side of the double well.

The initial state  $|i\rangle$  was chosen to be a Gaussian with minimal standard deviation at the top of the barrier between the two wells.

In all runs the state first delocalizes and fluctuates strongly. For the example shown in the figure, the expectation of the centre of mass  $\langle q \rangle$  first moves to the left, but eventually localizes in the right-hand well where it smoothly oscillates and relaxes toward the bottom of the well. Thanks to the quantum state localization, an initially symmetric state localizes on one side of the barrier, and the symmetry is broken. Note that it is the fluctuations that break the symmetry; on averaging over a large number of runs, the mean evolution is symmetric, with the localization taking place as frequently on one side as the other.

Using this as a model for the ammonia molecule, if it could be prepared in an excited symmetric vibrational state, its symmetry would soon be destroyed by its interaction with the environment. Furthermore, for energies at the top of the barrier or below, the molecule moves rapidly into one or other of the two configurations. This is much stronger than saying that one can no longer see interference between them.

In fact one can say more: it is very difficult to produce such molecules, because the interaction with the environment would have to be removed both at the time of preparation and during storage.



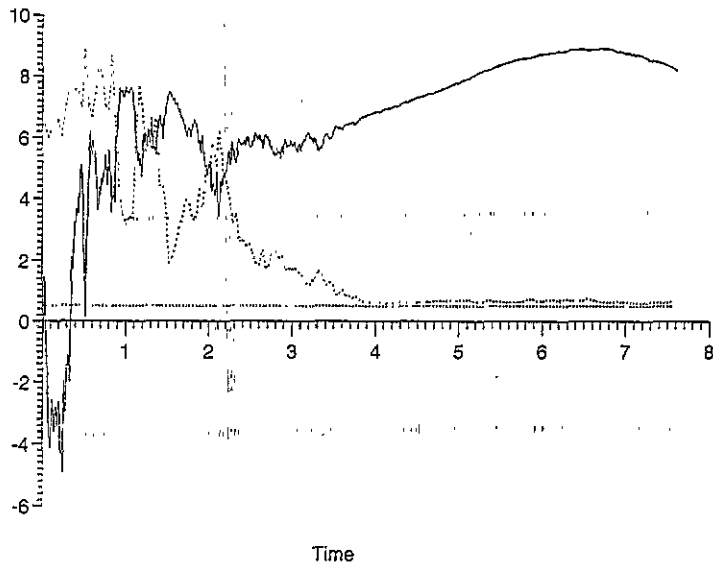
## 5. Phase-space localization

The centre of mass of a star or a football or a raindrop is localized in phase space. The quantum variances of position and momentum are both much smaller than small fluctuations due to external perturbations, so classical dynamics can be used.

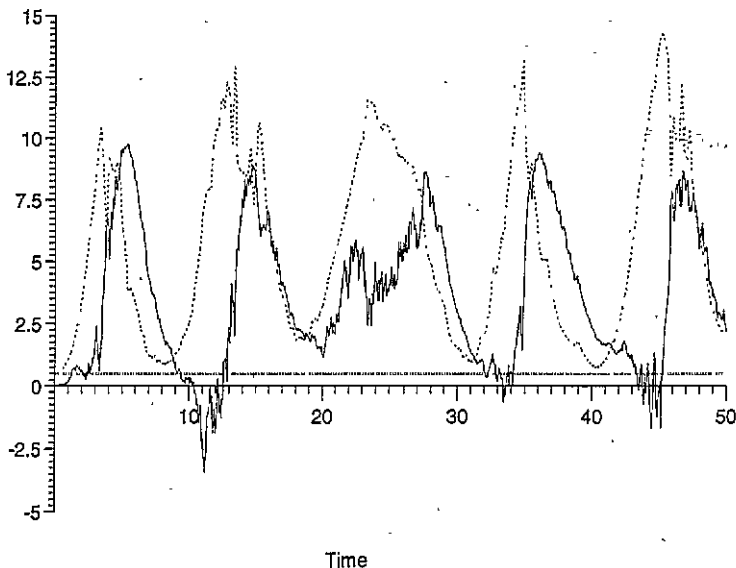
We have seen in section 2 that the localization in position or configuration space is a consequence of the fact that interaction potentials are localized in position space, but this does not explain the localization in momentum; indeed from the usual quantum viewpoint we would expect the position localization to produce a spread in momentum.

The localization in momentum is more subtle; the process takes place even for simple dissipative quantum systems, which illustrate the principles that apply at the macroscopic level. The simplest example is the forced damped oscillator treated in [2]. In that case an arbitrary initial state fluctuates, with a wide variance in both position and momentum, but tends eventually towards a coherent state in which both the position and momentum spread are reduced to small values with the minimum Heisenberg indeterminacy product of  $\hbar/2$ , as illustrated numerically in [1, figure 1]. Normally the localization is not nearly as efficient as this.

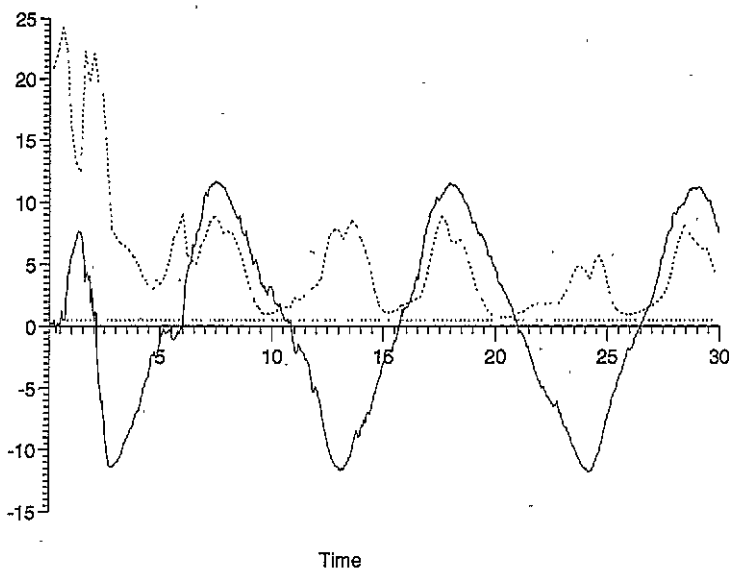
The process depends on the coupling of position and momentum by the dynamics. In this system, as in macroscopic systems, there is a continual interchange between kinetic and potential energies. As a result a wavepacket which is localized in the position or, more generally, the configuration variables, becomes a wavepacket that is localized in momentum. In state diffusion theory this transfer of localization is a consequence of the action of the usual Schrödinger terms, and not of the environment terms.



**Figure 4.** Single run of the quartic double well and position localization environment operator  $q/20$ , with initial symmetric state of low initial mean energy (see figure 1). The full line is the mean position  $\langle q \rangle$  and the upper dotted line is the Heisenberg indeterminacy product  $\Delta q \Delta p$ , both as a function of time. The horizontal dotted line is the minimum value of the product, which is  $\frac{1}{2}$ . After swinging towards the negative well, the system settles in the positive well, where it oscillates almost linearly and settles down to a state of close to minimum indeterminacy.



**Figure 5.** As figure 4, but with medium initial mean energy close to the top of the barrier. Each collision with the barrier produces both significant forward and significant backward scattering. The resultant Hamiltonian dispersion prevents phase-space localization for long times, as shown by the wild fluctuations of the indeterminacy product.



**Figure 6.** As figure 4, but with high initial mean energy. The oscillations in the quartic potential are highly nonlinear, yet there is phase space localization, as shown by the reduction in the indeterminacy product. However the phase space localization is not nearly so effective as for the linear oscillator.

So the localization in momentum takes place in two stages: first a localization in position due to interaction with the environment; and then the transformation of the position localization into momentum localization through the Schrödinger dynamics. Normally

the localization does not proceed as far as a minimum indeterminacy wavepacket, but such complete localization is quite unnecessary for the validity of classical mechanics in macroscopic systems. A more typical example of localization in a simple quantum system is given by the quartic oscillator of the previous section, with position localization represented by the environment operator  $q/20$ . The time dependence of the indeterminacy product  $\Delta q \Delta p$  is illustrated in figures 4–6 for various initial conditions, illustrated in figure 1.

When the mean energy is significantly less than the maximum barrier potential energy between the wells, the wavepacket settles down into one well or the other, and localizes approximately like a damped linear oscillator. When the mean energy is much greater than the barrier energy, the energy exchange mechanism produces a localization in momentum, in a fluctuating wavepacket whose indeterminacy product is significantly greater than the minimum. When the energy is near to the barrier energy, the wavepacket is split by collision with the top of the barrier into forward and backward scattered parts, resulting in a rapid Schrödinger dispersion, which is too strong for the localization process to overcome, so instead of localization there is a wild fluctuation.

For such simple systems the extent of the localization depends on the details of the dynamics, but for macroscopic systems, there are many localization processes, each due to a different environment operator, and partial barrier penetration is negligible, so in practice the environment localization always overcomes the Schrödinger dispersion.

## 6. Absorbing screens and detectors

In electric circuit theory, a distributed system like a transmission line or an aerial may be represented by a circuit with lumped constants to varying degrees of approximation. For example, an aerial may be represented by a single (dissipative) resistor, or a network of resistors, or by an infinite (Hamiltonian) array of inductances and capacitors, where each representation is an improvement on the previous one.

There are similar sequences of approximations in the state diffusion theory of localization whose validity depends on the localization theorems of [2]. A channel may have any number of states. In the simplest model it is represented by a single quantum state with projection operator  $P_k$  and localization with respect to this channel is represented by  $\beta^{1/2} P_k$  where  $\beta$  is a localization rate. A better representation may assign a finite number of states to the channel, and an environment operator  $P_k L_{kj}$  to represent each type of interaction of the system with its environment. By the dispersion entropy theorem applied to this channel and its complement, this set of operators will also localize the system into or out of the channel  $k$ . The representation may be made even more realistic by representing more of the system by explicit Hamiltonian terms. If these terms are to represent irreversible processes, then the state space must be infinite.

A particle of mass  $m$  propagating in free space meets a screen which can absorb it. Suppose the Hamiltonian operator of the state diffusion equation is given by

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \quad (6.1)$$

where the potential is non-zero only within the screen region, denoted by  $S$ . Because position plays an important role it is convenient to use a notation in which the wavefunctions  $\Psi(\mathbf{r}, t)$  which are solutions of the Schrödinger equation, and the wavefunctions  $\psi(\mathbf{r}, t)$ , which are solutions of the state diffusion equation, are represented explicitly as functions of  $\mathbf{r}$ .

Suppose that the projector  $P_1$  of the first channel projects onto the free space outside the screen and that  $P_2$  of the second channel projects onto the region of the screen. The ensemble probabilities  $p_1(t)$  and  $p_2(t)$  can be obtained from the solution of the Schrödinger equation, and in the common case when there is no reflection from the screen, they can be obtained from the solution of the equation in free space alone. Then

$$\alpha(t) = dp_2(t)/dt \quad (6.2)$$

is the absorption rate of the screen.

In the simplest state diffusion model, the space dependence is ignored; the channels 1 for free space and 2 for the screen are each represented by one state, and the transition from one to the other is represented by the time-dependent transition environment operator

$$L_\alpha = \alpha^{1/2} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}. \quad (6.3)$$

The interaction with the screen as an environment is represented by the projection environment operator  $P_2$  with a coefficient whose square is proportional to the effective absorption rate coefficient  $\beta$ , so it is

$$L_\beta = \beta^{1/2} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \quad (6.4)$$

Since the interactions with the screen are typically faster than  $\alpha$  and there are very many of them,  $\beta$  is typically very much greater than  $\alpha$ , so the transitions of the individual systems are rapid, even more rapid than the transitions illustrated in figure 6 of [1].

A better approximation represents the position dependence explicitly, by the wavefunction  $\psi(\mathbf{r}, t)$  which satisfies Schrödinger's equation inside and outside the screen, with normalization constants that are determined by the localization. Absorption or other effective interaction is represented by  $\beta^{1/2} P_2$ .

Before the wave  $\psi(\mathbf{r}, t)$  representing the incident particle strikes the screen, it is localized in free space. In the simplest two-state model, the subsequent behaviour resembles the radiative decay from the upper state of a two-state atom, as discussed in [1]. If  $\beta$  is large enough, then a sample system  $A$  remains in the free space channel until it makes a sudden irreversible transition to the screen channel. The only difference from the radiative decay model is that the transition rate is time-dependent. In the second model the same sudden transition takes place, but the space dependence outside the screen is represented explicitly. Although the representation by a single state of channels with many states is often adequate, it is oversimplified for some, like the quantum Zeno effect.

The second model is good enough to represent the localization between the screen and free space, but it is too simple to represent the details of what happens inside the screen. A third and better representation of the behaviour inside the screen is obtained by dividing the region  $S$  of the screen into many parts  $S_k$ , where each part contains absorbers. The corresponding absorptions are represented by projectors  $P_k$  with different coefficients  $\gamma_k$  to represent different absorption efficiencies. If the absorption is rapid by comparison with the Hamiltonian rates, the dispersion entropy theorem of [2] shows that a system will eventually localize in one of the regions  $P_k$ .

A very simple model of the process of detection is given by a modification of the first model of the absorbing screen. There are three channels. The first two are the same as for the absorbing screen, but there is a third channel, which is coupled to the second. The

amplitude in this channel corresponds to the detection of the particle. The third channel is coupled to the second by an irreversible transition environment operator of the form (6.3), and it is localized by a projection operator of the form (6.4). In this model it is assumed that every particle is detected. To represent the inefficiency of real detectors a fourth channel is required, to represent the undetected particles, with a transition operator from channel 2. An example of a simple model of detection is given in the next section on the photographic latent image.

For a particle to be detected, it is necessary for it to change the state of the detector. This may be achieved by absorption, but that is not necessary, and the localization which takes place due to *any* change in the state of any system like a detector can be modelled as shown above for an absorbing screen.

## 7. Photographic latent image

A photographic latent image is an example of a single-photon detector which clearly distinguishes the formation of a robust microscopic record of a quantum event, and the subsequent amplification of that record to the macroscopic level [13]. The first is the formation of the latent image, which is described in this section, and the second occurs during production of the negative, which may take place weeks or months later.

We present the state diffusion picture of the standard theory of Gurney and Mott [14, 15] for the formation of the latent image. According to this theory each effective photon leaves a record as a result of a two-stage process.

In the first stage the photon enters a crystal of silver bromide embedded in gelatine and ejects an electron from the negative bromine ion of a polar silver bromide molecule:



The photon is not detected if the electron returns to the original silver bromide, but immediately after ejection it has too much energy to do so [15].

Instead, in the second stage, it diffuses inside the local silver bromide crystal to its surface, where it is trapped by the  $\text{Ag}^+$  of the polar silver sulphide molecules that are located there. These trapped electrons attract further  $\text{Ag}^+$  ions, which form the latent image, as shown in figure 7.

This is how the process are usually described in the general literature, as if they were classical, but in fact the processes are quantal, as they must be for the detection of individual photons. The quantum description differs from the classical description in that the photons, atoms and electrons are represented by state vectors or wavefunctions. This holds both for the orthodox quantum picture and for the state diffusion theory. However, in the orthodox picture it is meaningless to ask when the events described actually take place, whereas in the state diffusion picture it does make sense, for it allows us to visualize what is happening to the state vector, which continuously diffuses and localizes because of the interaction with the environment.

A simplified model of these processes consists of dividing the state space of the photon and molecules into channels, each represented by a single state, as in the detector theory of the previous section:

*channel 1*—the incident photon state together with a silver bromide molecule, and a silver sulphide molecule,  $|1\rangle$ ;

*channel 2*—states with the photoelectron moving inside the crystal,  $|2\rangle$ ;

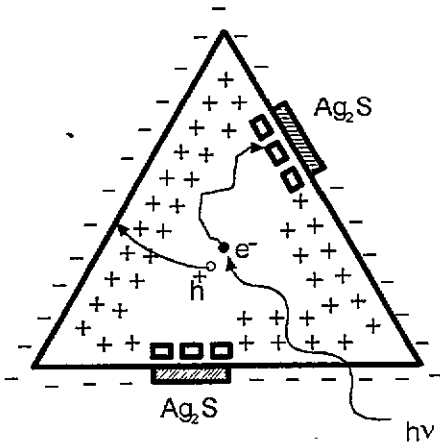


Figure 7. The processes leading to the formation of a latent image in photography. See text.

*channel 3*—states with the photoelectron captured by a silver ion of the silver sulphide molecule, [3].

The first step of the process is then modelled by a transition environment operator from state  $|1\rangle$  to state  $|2\rangle$ . The coefficient depends on the light intensity. The localization in the second channel is represented by the projector onto state  $|2\rangle$ , with an appropriate coefficient, which is clearly very large, to represent all the interactions of the electron and ion with the crystal. The second step is described by the quantum state diffusion from state  $|2\rangle$  to state  $|3\rangle$ . In the Gurney and Mott theory as well as in our model this second step is irreversible, dissipative and stochastic; it takes place at the microscopic level. Again the localization is represented by a projection operator, this time onto state  $|3\rangle$ . Typically there is a sudden transition from channel 1 to channel 2 and another from channel 2 to 3.

Note that if the incident photon were delocalized, it could excite two  $\text{Ag}^+\text{Br}^-$  molecules coherently, thus producing a coherent superposition of two photoelectrons. In our model the state diffusion reduces this superposition during the second step of the process, resulting in a single photoelectron that can record the classical event 'absorption of a photon'.

The model could be made more realistic as described in the previous section.

During the development of the photographic plate, crystals containing sufficient AgS molecules in state  $|3\rangle$  are darkened. State  $|3\rangle$  acts as a strong catalyser allowing crystals containing them to become distinguishable by the naked eye. There can be a period of weeks or months between the formation of the latent image and the process of development. The first stage is the quantum detection which takes place entirely at the microscopic level. The second stage is the amplification which is required to produce a macroscopic effect. For this example, as for DNA [13], the two processes are clearly separated.

## 8. Radioactive decay

This illustrates how the Schrödinger equation summarizes the statistics of the detailed behaviour of individual systems in the state diffusion model. Because position plays an important role it is convenient to use a notation in which the wavefunctions are represented explicitly.

A radioactive nucleus  $A$  with vector coordinate  $r$  decays into two decay products  $A_1, A_2$  with vector coordinates  $r_1, r_2$ . The two-vector  $\chi$  represents the state of decay, equal to  $\chi_0$  for the radioactive atom and the orthogonal  $\chi_{12}$  for the decay products, where

$$\chi_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \chi_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (8.1)$$

If the radioactive atom is formed at time  $t = 0$ , then the solution of the Schrödinger equation for later times is represented to a good approximation by

$$\Psi(r, t) = \chi_0 \psi_c(r) e^{-\alpha t/2} + \chi_{12} \psi_c((m_1 r_1 + m_2 r_2)/(m_1 + m_2)) \psi_r(r_2 - r_1, t) \quad (8.2)$$

where subscripts  $c$  and  $r$  refer to centre-of-mass and relative motion wavefunctions, and  $\alpha$  is the exponential decay rate.

In the state diffusion model, this solution of the Schrödinger equation only represents the actual behaviour of the radioactive nucleus and its decay products in those extremely rare or non-existent conditions for which neither the nucleus nor either product interacts with its environment. As soon as interaction takes place there is diffusion which leads to localization. This can be represented in increasing detail by a succession of approximations.

In the simplest and crudest approximation, the spatial dependence is ignored, and the radioactive decay is represented by an environment operator which is transition operator in the  $\chi$ -space:

$$L_\alpha = \alpha^{1/2} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad (8.3)$$

which produces the exponential decay of the radioactive atom. The interactions of the radioactive atom and the decay products with their environments are represented by projection environment operators onto the corresponding states, with coefficients that represent the strength of the interactions, which normally act much more quickly than the radioactive decay rates:

$$L_\beta = \beta^{1/2} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \quad (8.4)$$

This is identical to the two-state version of the quantum cascade illustrated in [1, figure 6]. The model state vector diffuses in the close neighbourhood of  $\chi_0$  until it makes a relatively sudden transition to the decayed state.

In a better approximation the decay of the nucleus is represented by its Hamiltonian  $H$ , which, if acting on its own, would give the solution (8.2). The decay is thus represented by the Hamiltonian, and does not require an additional environment term. The interaction of the radioactive nucleus with its environment is represented as before, but the position dependence of the interactions of the decay products is represented explicitly by projection environment operators  $P_k$  onto the regions of space where the interaction can take place, with coefficients  $\gamma_k$  whose magnitude is obtained from the effective interaction rates.

## 9. Discussion

We have applied state diffusion theory to a wide variety of physical processes using models and approximations similar to those of elementary classical electric circuit theory. We

have not yet carried out the detailed analyses of the physics of the interactions with the environment which lead to the localization, but that could be done by calculations of rates similar to those that are used for the density operator theory. Such a state diffusion analysis would be equivalent to deducing the behaviour of a classical resistor from the detailed behaviour of conduction electrons within it, which is much more difficult than the circuit theory itself.

A general pattern emerges from these studies. When a diffuse quantum wave of a simple or compound system like an electron, atom or molecule moves into an absorbing solid or liquid, the localization rate is determined by rate constants within the absorber, which are normally much faster than the typical mean transition rate of the particle into the absorber. Consequently the transitions for the individual systems of the ensemble take place much more rapidly than the mean rate of absorption [1, figure 6], and resemble the instantaneous quantum jumps which are the basis of alternative stochastic state-vector simulations, see [1, 2, 16]. This applies not only to atomic or nuclear transitions, including radioactive decay, but also, for example, to the absorption of a particle by a screen.

Although the general theory applies to localization with respect to arbitrary dynamical variables, the finite range in position space of effective interactions between particles gives a particular importance to position localization, which does not have to be assumed, as in the Ghirardi–Rimini–Weber theory [4, 6]. The position localization of individual particles leads to configuration localization of compound systems like molecules interacting with their environment. The environment determines the Galilean frame of the localization. The stability of chiral molecules and the associated symmetry breaking are almost trivial consequences of the position localization of the atoms of the molecule. The principles are illustrated by the behaviour of a quartic double-well oscillator in various approximations and with various environment operators. These are also used to demonstrate phase-space localization, which is observed everywhere for macroscopic systems.

Of particular importance is the study of the formation of the latent image in photography, for this is not only the most common quantum detection technology, but it also shows unequivocally that amplification up to the macroscopic level is quite unnecessary for the formation of a permanent classical record of a quantum event, by contrast with the example of the pointer, which is so often used.

## Acknowledgments

We would like to thank C Dainty and F Walther for guides to the photographic literature and R Passy for help with the figures. ICP would like to thank the UK Science and Engineering Research Council for financial support, and, for their hospitality and stimulating discussions, the Quantum Optics Group at Imperial College London and Theoretical Physics at CERN Geneva.

## References

- [1] Gisin N and Percival I C 1992 *J. Phys. A: Math. Gen.* **25** 5677–91; 1992 *Phys. Lett.* **167A** 315–8
- [2] Gisin N and Percival I C 1993 *J. Phys. A: Math. Gen.* **26** 2337
- [3] Bell J S 1987 *Schrödinger, Centenary of a Polymath* ed C Kilmister (Cambridge: Cambridge University Press); 1987 Reprint *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press)
- [4] Ghirardi G-C, Rimini A and Weber T 1986 *Phys. Rev. D* **34** 470–91



- [5] Gisin N 1989 *Helv. Phys. Acta* **62** 363–71
- [6] Ghirardi G-C, Pearle P and Rimini A 1990 *Phys. Rev. A* **42** 78
- [7] Diósi L private communication
- [8] Primas H 1983 Chemistry, quantum mechanics and reductionism *Perspectives in Theoretical Chemistry* (Berlin: Springer)
- [9] Zurek W H 1982 *Phys. Rev. D* **26** 1862–80
- [10] Quack M 1989 *Angew. Chem. Int. Ed. Engl.* **28** 571
- [11] Amman A 1991 *J. Math. Chem.* **6** 1–15
- [12] Harris R A and Stodolsky L 1981 *J. Chem. Phys.* **74** 2145–56
- [13] Percival I C 1991 *Nature* **351** 357
- [14] Gurney and Mott N F 1938 *Proc. R. Soc.* **164** 151  
Mott N F and Gurney R W 1950 *Electronic Processes in Ionic Crystals* (Oxford: Clarendon)
- [15] Hamilton J F and Urbach F 1964 *The Theory of the Photographic Process* 3rd edn, ed C E K Mees and T H James, ch 5, pp 87–119  
Glaflkidès P 1987 *Chimie et physique photographiques: Tome 1* (Paris: Editions de l'Usine Nouvelle)
- [16] Gisin N, Knight P L, Percival I C, Thompson R C and Wilson D C 1992 *Preprint*